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**STERILE NEUTRINOS IN A 6X6 MATRIX APPROACH**

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Quark-lepton symmetry invites consideration of the existence of sterile neutrinos. Long ago, we showed that this approach predicts large neutrino mixing amplitudes. Using a Weyl spinor approach, we show, in an analytic example, how this, and pseudo-Dirac pairing, can develop within a reduced rank version of the conventional see-saw mechanism, from small intrinsic mixing strengths. We show by numerical examples that mixing of active and sterile neutrinos can affect the structure of oscillations relevant to extraction of neutrino mixing parameters from neutrino oscillation data.

*Keywords:* McKellar; Neutrinos; Sterile; Mass; Festschrift.

**1. Quark-Lepton Symmetry Is Our Basis**

In the Standard Model (SM) as first formulated, there were no neutrino mass terms as no right-chiral projections of Dirac neutrino fields were known to exist. Excluding them left no means to produce Dirac neutrino mass terms and Majorana mass terms required introduction of either non-renormalizable terms in the Lagrangian or new scalar fields with unit weak isospin.

However, the formulation of Grand Unified Theories (GUTs) in the mid-70's,  $SU(5)$  in particular,<sup>1</sup> made clear that the fundamental degrees of freedom were not Dirac bispinors but two-component Weyl spinors. Later developments in supersymmetry and supergravity amplified this contention. In the Weyl spinor basis, all known fermions, except the neutrinos, appeared in left- and right-chiral pairs  $((\frac{1}{2}, 0)$  and  $(0, \frac{1}{2})$  irreps under the Lorentz group). The pairing, along with equal mass terms, were necessary to allow construction of Dirac bispinors which could satisfy the known (to high accuracy) conservation of electric and color charges. Thus, right-chiral partners for the known neutrinos were not required, but, to some of us, at

least, seemed strongly invited, especially as the successes of quark-lepton symmetry grew over the following decade: charm,<sup>2</sup> and then after the discovery<sup>3</sup> of the  $\tau$ -lepton, the  $b$ -quark<sup>4</sup> and eventually the  $t$ -quark.<sup>5</sup>

## 2. See-Saw Mechanism

At Los Alamos, a number of researchers and visitors, including Stephenson, Slansky, Ramond and Gell-Mann,<sup>6</sup> recognized that the right-chiral fields were unconstrained, even in  $SU(5)$ , in the possible Majorana (or as we prefer to say here, Weyl) mass term possible – the mass could be as large as the GUT scale ( $M \sim 10^{16}$  GeV). Furthermore, this, combined with now normal (order quark or charged lepton) Dirac mass terms ( $m \sim 10^{\pm 3}$  GeV) that should appear, would produce eigenstates with very small Majorana masses ( $\sim m^2/M$ ) and that were almost purely left-chiral neutrinos, that is, those that participate in the weak interactions. As a bonus, the known Cabibbo-Kobayashi-Maskawa (CKM) mixing<sup>7</sup> between quark mass and weak interaction eigenstates strongly suggested that similar physics should develop in the lepton sector, producing the long-conjectured oscillation of neutrinos<sup>8</sup> (although between different flavors rather than between particle and antiparticle as originally suggested).

### 2.1. *An Early Effort*

In the absence of any credible detailed conjectures as to the structure of the mass matrices in the lepton sector (although there were a plethora of papers about what would now be called "textures"), we<sup>9</sup> carried out a Monte Carlo study (popular more recently in considering the possible values of the multiple parameters in supersymmetric theories) using random choices for mass matrix entries, allowing for CKM-like mixing of the quarks in the Dirac mass sector of the leptons.

The results were rather astonishing. As Fig. 1, taken from that work shows, Cabibbo mixing is favored between the first two "generations" and even larger mixing is highly probable between the second two, depending upon how extreme the third generation differs in mass. (Note that we did not have the temerity to consider an extreme as radical as actually occurs in the quark sector.) The mixing between the first and third "generations" is smaller, but non-vanishing. At the time, the only potential evidence for neutrino mixing was the intermediate result of the Davis experiment,<sup>10</sup> which was considered highly suspect, although it was later confirmed quite precisely.<sup>11</sup> We used our result to support mounting of neutrino oscillation experiments, saying that the mixing might well be large, although we could not predict the scale of the oscillation length.

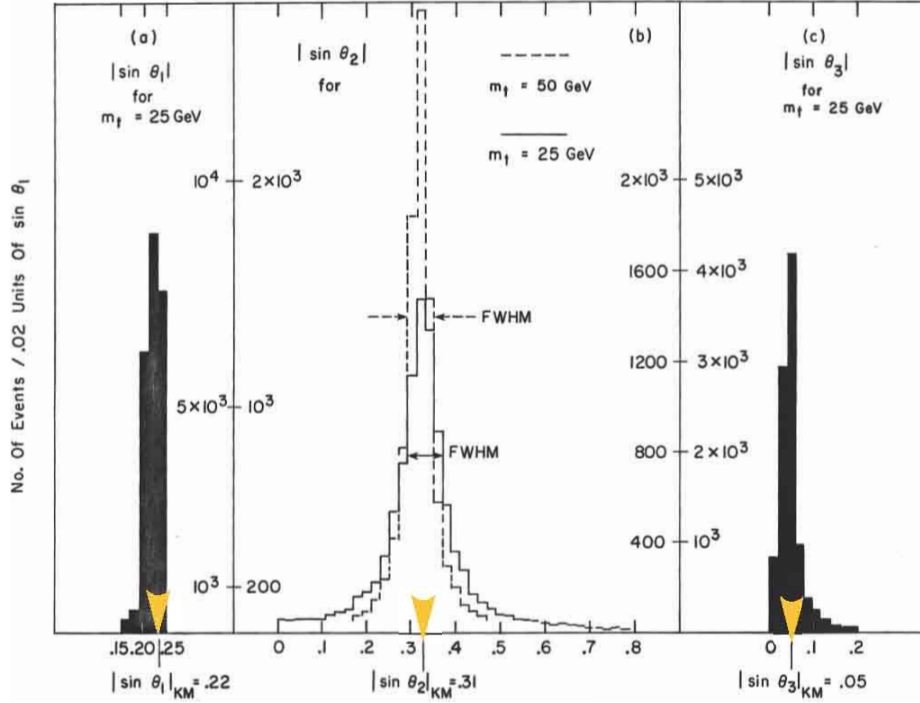


Fig. 1. Mixing angle distributions for random mass matrix entries from Ref. 9.

### 3. Weyl spinors

Since active neutrinos have only two basic states (as opposed to the four of a Dirac bispinor), they can be efficiently described in terms of Weyl spinors. We present the Lagrangian, equations of motion, and solutions for massive Weyl spinors, then show the relation to Majorana and Dirac constructs.

#### 3.1. Lagrangian Density for Massive Weyl Spinors

Let the Grassman-valued field variable,  $\phi$ , represent a left-chiral  $(\frac{1}{2}, 0)$  irrep of the Lorentz Group. Then

$$\mathcal{L}_L = \frac{1}{2} \phi^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \phi + \frac{1}{2} i m (\phi^T \sigma^2 \phi + \phi^\dagger \sigma^2 \phi^*) \quad (1)$$

where  $\overleftrightarrow{\partial} \equiv \overrightarrow{\partial} - \overleftarrow{\partial}$  and  $\sigma^\mu = (1, \sigma^i)$  with  $\sigma^i$  the Pauli matrices. Under a Lorentz transformation with parameters,  $\omega_\mu$ ,

$$\phi \rightarrow e^{-\frac{i}{2}(\sigma^\mu \omega_\mu)} \phi \quad (2)$$

For a right-chiral  $(0, \frac{1}{2})$  irrep of the Lorentz Group, we need only make the substitution:

$$\mathcal{L}_R : \sigma^\mu \rightarrow \bar{\sigma}^\mu = (1, -\sigma^i) \quad (3)$$

in Eq. 1 to acquire the relevant Lagrangian.

Because mass terms must couple left-chiral  $(\frac{1}{2}, 0)$  and right-chiral  $(0, \frac{1}{2})$  irreps of the Lorentz Group, it is apparent that

$$\chi = \sigma^2 \phi^* \quad (4)$$

must be in a right-chiral  $(0, \frac{1}{2})$  irrep, as can be seen by applying the boost in Eq. 2 to  $\phi$  irrep and then applying the commutation rules for the Pauli matrices to the construction in Eq. 4. This will be relevant shortly.

### 3.2. Equations of Motion and Form of Solutions

Writing  $\phi$  out explicitly as a two component column spinor,

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad (5)$$

the equations of motion for the components become

$$\begin{aligned} \partial_t \phi_1 - \partial_z \phi_1 - (\partial_x - i\partial_y) \phi_2 &= -m\phi_2^* \\ \partial_t \phi_2 + \partial_z \phi_2 - (\partial_x + i\partial_y) \phi_1 &= +m\phi_1^* \end{aligned} \quad (6)$$

Defining  $\theta = Et - \vec{p} \cdot \vec{x}$  and  $p_\pm = p_x \pm ip_y$ , we find the complex conjugate pair of solutions,  $\phi_-$  and  $\phi_+ = \phi_-^*$  to have the form

$$\phi_- = \begin{pmatrix} F e^{-i\theta} \\ -\frac{p_+}{E-p_z} F e^{-i\theta} - i\frac{m}{E-p_z} F^* e^{+i\theta} \end{pmatrix} \quad (7)$$

where  $F$  is a Grassman-valued constant.

### 3.3. Majorana and Dirac Bispinors

A Majorana bispinor is simply a redundant representation of the Weyl spinor above. In the Wigner-Weyl representation for the bispinor, we make use of the transformation in Eq. 4, to construct

$$\Psi_M = \begin{pmatrix} \phi \\ e^{i\eta} \sigma^2 \phi^* \end{pmatrix} \quad (8)$$

where the phase  $\eta$  can be chosen as  $0, \pm\pi/2$  or  $\pi$  for later convenience. The field  $\Psi_M$  has a Lagrangian that can be put into Dirac form with mass  $m$ .

To construct a Dirac bispinor, two independent  $(\frac{1}{2}, 0)$  irreps must be invoked, which we label suggestively as  $a$ , or active neutrino in the SM and  $s$ , for sterile neutrino in the SM. (Except for the  $U(1)$  factor, these terms apply to the left- and right-chiral parts of the charged fermions as well.) Thus,

$$\Psi_D = \begin{pmatrix} a \\ -\sigma^2 a^* \end{pmatrix} + \imath \begin{pmatrix} s \\ -\sigma^2 s^* \end{pmatrix} = \Psi_a + \imath \Psi_s \quad (9)$$

where the phase choices have been made so that if  $\Psi_a$  and  $\Psi_s$  have the same mass value  $m$ , then (as can be seen from Eq. 1)  $\imath \Psi_s$  has mass value  $-m$  and a  $45^\circ$  rotation in the basis space will explicitly display  $m$  as a Dirac mass. (See Eq. 10 below.)

The rest states of two such independent spinors (see Eq. 7), with independent Grassman constants  $F$  and  $G$ , can be combined (with  $F = -G$  and  $H$  the sum) to produce the familiar form of a spin-up Dirac particle in the Pauli-Dirac representation, *viz.*

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} H e^{-\imath m t} \\ 0 \\ H e^{-\imath m t} \\ 0 \end{pmatrix} = \begin{pmatrix} A e^{-\imath m t} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (10)$$

where again,  $F, G, H$  and  $A$  are all Grassman-valued constants.

#### 4. Beyond the Simple See-Saw: Reduced Rank

The see-saw mechanism, as originally invoked, assumed, for simplicity, that the Majorana mass matrix of the right-chiral fields, represented as left-chiral but sterile neutrinos, is proportional to the unit matrix. Since then, many different "textures" for mass matrices of the fundamental fermions have been conjectured. We<sup>12</sup> (and others<sup>13</sup>) have studied the possibility that the rank of this so-called right-handed mass matrix is less than three.

With quark-lepton symmetry, the mass matrix structure consists of four three-by-three blocks: The  $(1, 1)$  block describes the Majorana masses of the active neutrinos and must vanish in the absence of a triplet Higgs field. The  $(1, 2)$  and  $(2, 1)$  blocks describe Dirac mass terms ( $m$ ) that couple the active and sterile Weyl spinor neutrino fields. If we take them to be diagonal for the moment, this defines the flavor of each sterile neutrino field as a partners of a particular active neutrino. Finally, the  $(2, 2)$  block describes the Majorana masses of the sterile neutrinos. In this basis, we can initially, for a rank one sterile mass matrix, set all of the entries to zero except for the  $(3, 3)$  element of the  $(2, 2)$  block, which we label  $M$ .

These alignments are unrealistic, of course, so we carry out two sets of rotations: One corresponds to moving the vector  $(0, 0, M)$  in the sterile "flavor" space away from the "3" axis with the standard angles,  $\theta$  (from the 3-axis) and  $\phi$  (in the 1-2 plane). In addition, we allow for CKM-like mixing in the Dirac mass matrix sector.

Absent CP-violation, this accounts for all of the possible mixings among the six fields and their corresponding particle states.

We found in this system, that there is a wide range of parameters over which the mass eigenstates form into a very light, mostly active neutrino as in the conventional see-saw, a very heavy (under the assumption that  $M \gg m$ ) mostly sterile neutrino, again conventionally, and two pairs of "pseudo-Dirac" neutrinos, the the sense of Wolfenstein.<sup>14</sup> The resulting mixing amplitudes among the active neutrinos are very large to maximal.

#### 4.1. Analytic Analysis of Two Flavor Case

In order to understand this better, we solve analytically the simpler two flavor case. The eigen equation for this system is, after rotation from exact alignment (and ignoring the analog of CKM mixing),

$$\mu_i \Phi_i = \begin{pmatrix} 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_3 \\ m_1 & 0 & Ms^2 & Mcs \\ 0 & m_3 & Mcs & Mc^2 \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_i \\ \gamma_i \\ \delta_i \end{pmatrix} \quad (11)$$

where the  $\Phi_i$  are four component column vectors with entries as indicated on the RHS of the equation, and  $s = \sin\theta$  and  $c = \cos\theta$  where  $\theta$  is the misalignment angle between active and sterile flavor spaces.

McKellar showed that the eigenvalues of this system are

$$\begin{aligned} \mu_1 &= +m_0 - \frac{a^2}{M} - \frac{a^2}{m_0 M^2} (m_0^2 - \frac{a^2}{2} - b^2) \\ \mu_2 &= -m_0 - \frac{a^2}{M} + \frac{a^2}{m_0 M^2} (m_0^2 - \frac{a^2}{2} - b^2) \\ \mu_3 &= -\frac{b^2}{M} + \mathcal{O}(M^{-3}) \\ \mu_4 &= M + \frac{2a^2 + b^2}{M} + \mathcal{O}(M^{-3}) \end{aligned} \quad (12)$$

where

$$\begin{aligned} m_0^2 &= m_1^2 \cos^2\theta + m_3^2 \sin^2\theta \\ a &= \frac{(m_1^2 - m_3^2) \cos\theta \sin\theta}{m_0 \sqrt{2}} \\ b &= \frac{m_1 m_3}{m_0} \end{aligned} \quad (13)$$

Note the low and high mass see-saw pair,  $\mu_3$  and  $\mu_4$ , and the psuedo-Dirac pair,  $\mu_1$  and  $\mu_2$ , which would form a single Dirac neutrino to this order if it happened that  $a = 0$ .

The eigenvector components for these solutions satisfy

$$\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\beta_3}{\alpha_3} = \frac{m_1}{m_3} \cot \theta \quad (14)$$

and

$$\frac{\gamma_i}{\alpha_i} = \frac{\mu_i}{m_1} ; \quad \frac{\delta_i}{\beta_i} = \frac{\mu_i}{m_3} \quad (15)$$

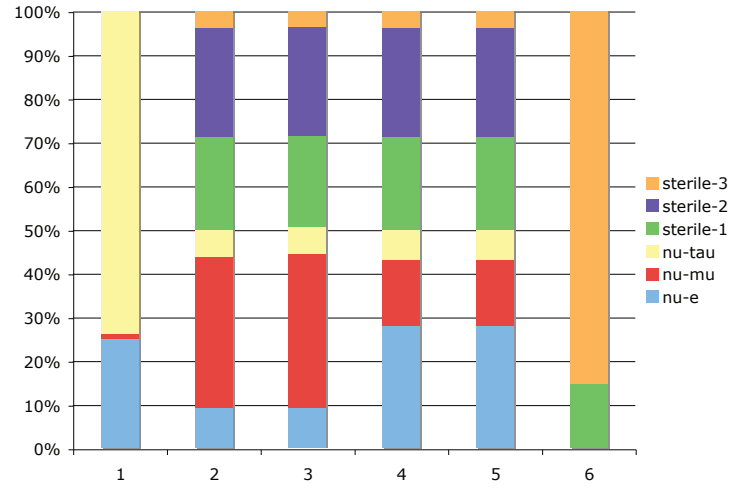


Fig. 2. Three flavor example with parameters chosen to demonstrate large flavor mixing and two pseudo-Dirac pairs ([2,3] and [4,5]). The bands indicate the relative amplitudes of each active and sterile flavor in each eigenmass state enumerated along the baseline.

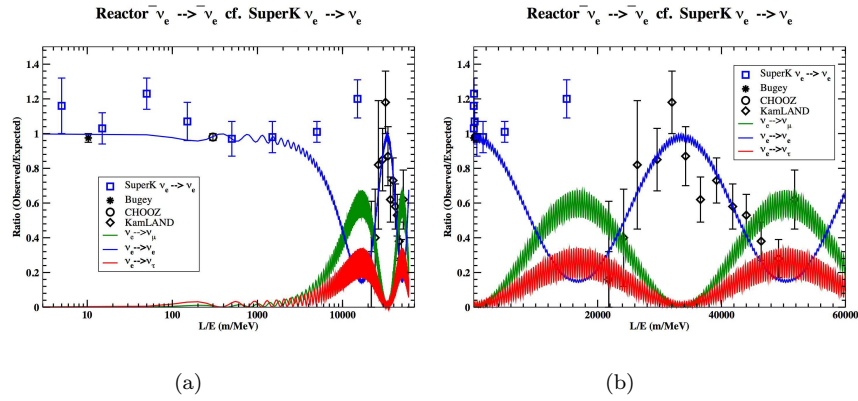


Fig. 3. Electron antineutrino disappearance vs. ratio of distance from source divided by neutrino energy compared with experimental data. (a) Logarithmic plot. (b) Linear plot.

Hence for relatively large  $M$  and small  $\theta$ , eigenstates 3 and 4 are almost purely active and sterile respectively, while  $\mu_1 \sim \mu_2 \sim m_1$  and the states with these two eigenvalues will have large components of both active flavor states when

$$\frac{m_3}{m_1} \sim \cot\theta$$

that is, the terms in Eq. 14 are of order one.

#### 4.2. Results for Three Flavor Case

An analytic demonstration is not so easy to provide in the three flavor case, but a similar result, with two pseudo-Dirac pairs and large flavor mixing, is shown in Fig. 2 as an example case.

In Fig. 3(a), we show the result of a specific choice of parameters for electron antineutrino disappearance, which is consistent with the results of atmospheric and reactor experiments. Within uncertainties, the last two high points of the atmospheric

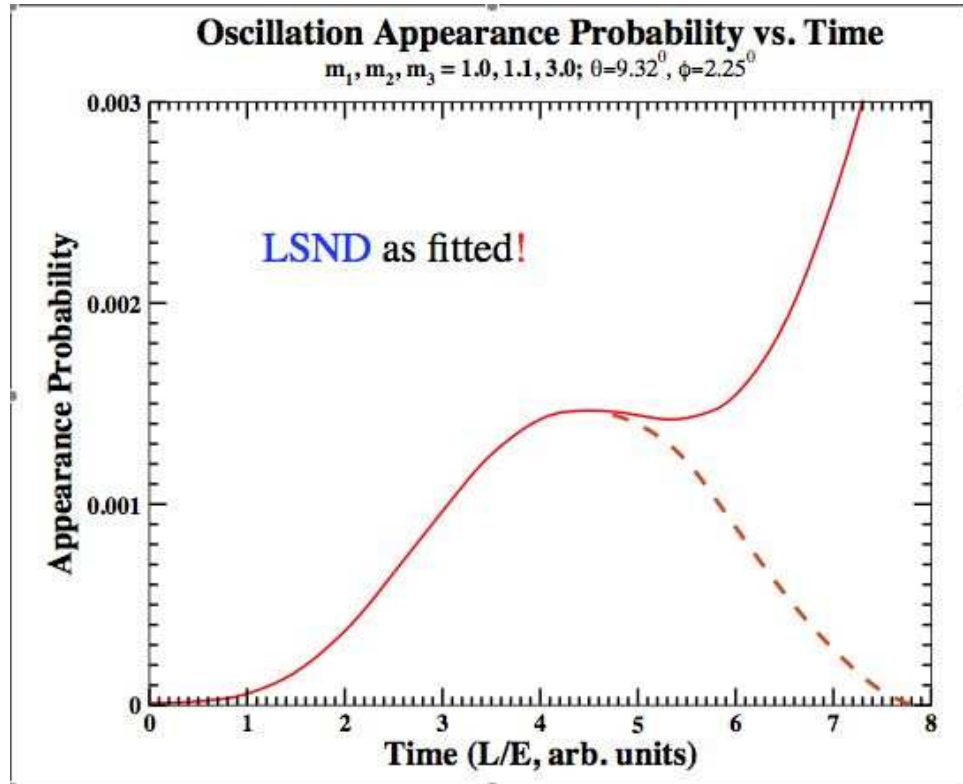


Fig. 4. Comparison of electron neutrino appearance vs. ratio of distance from source over neutrino energy as predicted by our lower rank, six-channel mixing model and the functional form (dashed line) assumed in the fit made by the experimental group.



(SuperKamiokande<sup>15</sup>) experiment are consistent in our parametrization with feed-through into electron neutrinos from the disappearance of muon neutrinos, also seen in that experiment. Fig. 3(b) focuses in detail on the region studied by the KamLAND experiment.<sup>16</sup>

In Fig. 4, we show the shape of the electron neutrino appearance function appropriate to the LSND experiment<sup>17</sup> and contrast this with the shape of the fitting function actually used (with the dashed extension corresponding to the simple sinusoidal function of two-channel mixing). This demonstrates that incorrect parameters may be extracted from experiments by not fitting directly to the full panoply of possibilities allowed by the three known flavors of neutrinos.

Finally, in Fig. 5, we show an example of how finite resolution, particularly in the neutrino energy, affects the oscillation pattern that is observed. The rapid oscillation between the initial muon neutrino and strongly mixed tau neutrino is smeared into

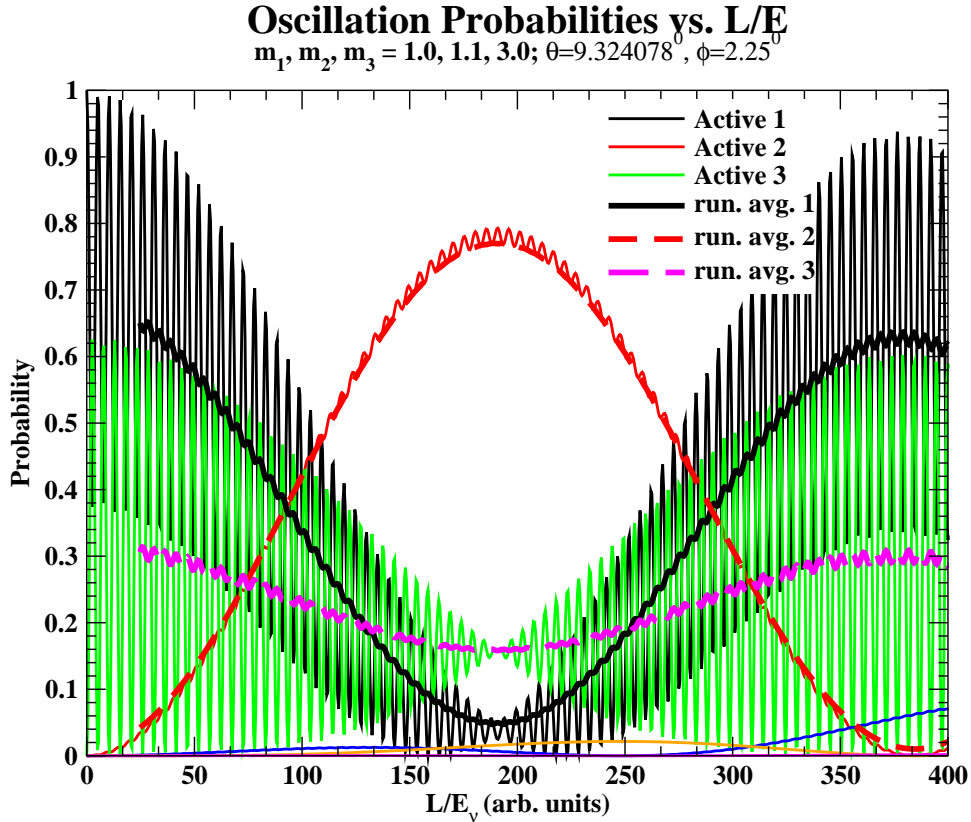


Fig. 5. Effect of finite resolution on disappearance and appearance as a function of time in the neutrino rest frame (or equivalently, ratio of distance from source to neutrino energy). The average of the disappearance of the initial neutrino flavor has approximately the expected shape for a much longer wavelength mixing than is seen at high resolution. See text for more discussion.

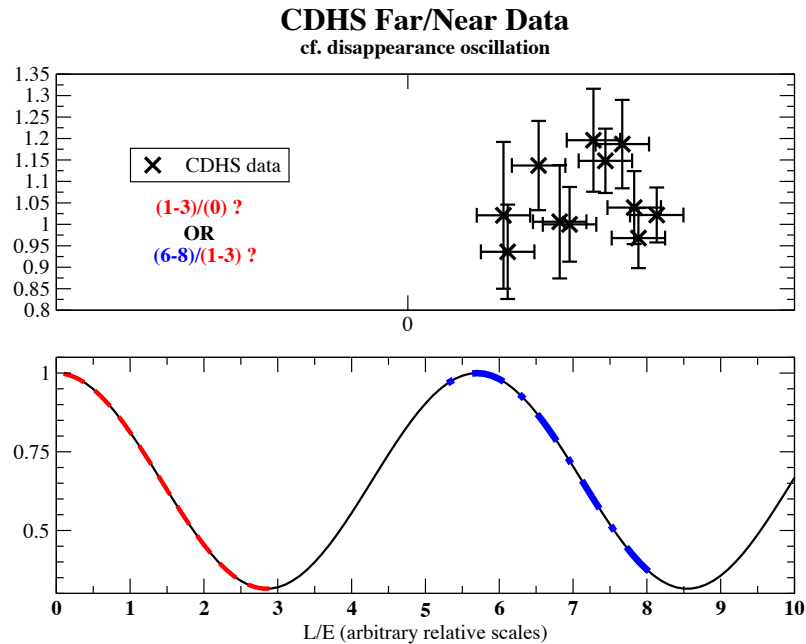


Fig. 6. Illustration of how the distance-corrected ratio of events in a far detector to those in a near detector can exceed unity if a very short wavelength oscillation affects the rate in the near detector and comparison with CDHS results.<sup>18</sup> See text for more discussion.

a much longer wavelength average muon neutrino disappearance. Without explicit detection of tau neutrinos, it is difficult to discern that their appearance is not following the expected, simple sinusoidal two channel appearance structure, but rather is almost constant, falling slightly while electron neutrinos actually appear in a growing fashion. Fig. 4 is a magnification of a very small region near the origin of this plot, and it is with the parameters noted here that the exceptional points referred to in Fig. 3 are explained.

#### 4.3. *One Detector or Two?*

A final note in passing: A vigorous debate that continues in the experimental community concerns the question of whether spectral distortion or two detectors at different distances from the same neutrino source affords the better means to observe and study neutrino oscillation phenomena. The latter is, of course, generally more expensive, so one might be inclined to think it is also more valuable. A curious result from the CDHS experiment, however, demonstrates that one must first be certain that the near detector is so close that no oscillations at all have taken place

by the time the beam arrives at that detector. In the CDHS results reported,<sup>18</sup> the flux in the farther detector is generally larger than in the near detector. Since this violates unitarity, it allows for a very stringent limit on neutrino disappearance. However, as we illustrate in Fig. 6, the excess can also be due to the near detector reacting to the first wave of oscillation disappearance, with the far detector appearing to have a larger ( $L^2$  corrected) flux by being at a slightly different phase in the oscillation wave.

## 5. Conclusion

We draw several conclusions from the above, not all of them warranted.

- There may well be 5 independent neutrino mass differences that must be fit to neutrino oscillation experimental data.
- Analyses of oscillation data in terms of  $2 \times 2$  mixing can miss significant physics and even lead to extraction of invalid parameter values.
- A global, multichannel analysis is essential before firm conclusions can be reliably drawn regarding neutrino masses and mixing parameters.
- With the  $\sim$ eV mass scales we have examined, neutrinos can contribute significantly to the Dark Matter in the Universe.

This paper includes work done with Jerry Stephenson and Bruce McKellar over many years. It has been our great pleasure to work with Bruce and we hope that, as soon as he retires, he will have a lot more time to work with us!

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